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Critical Review

Activity Theory and the Utilisation of the Activity System according to the Mathematics Educational Community

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Abstract*: The* ***activity system*** *is the unit of analysis, the analytic lens at which learning is perceived, in Cultural-Historical Activity Theory (CHAT, hereafter AT). Pictorially, the* ***activity system*** *is represented as a triangular structure of interconnected relationships formed between its six components (subject, object, community, instruments, rules, and division of labour). The significance of the* ***activity system*** *hinges on its capacity to define and analyse* ***contradictions****.* ***Contradictions*** *is a hallmark concept within AT defined as "sources of change and development" driving the* ***activity system*** *(Engeström, 2001, p 137). With an aim to understand the* ***activity system****, I first give a brief overview of its historical development, followed by an analysis of the ways in which the mathematics educational community has operationalised this unit of analysis. I argue that the* ***activity system*** *ought to be methodologically used in ways that allow researchers to broaden the exploration of* ***contradictions****. In particular, I suggest that nesting the activity system within broader institutional and cultural-historical contexts provides an innovative view adept for the analysis of* ***contradictions****.*

Contextualisation

A key objective of the author's doctoral studies and of the educational researcher's work is to shed light into learning processes. To this aim, educational researchers of mathematics have mainly drawn upon sociocultural and situated theories to describe learning as changes, shifts, or transformations in an individual or a group of individuals (Boaler, 2002; Anderson *et al*, 2000). One of the most important differences in these theories is the unit of analysis - that is, the analytic lens at which the learning process is perceived. The unit of analysis in Sociocultural Theory is the *mediated action* (Vygotsky, 1981). This theory describes learning as a developmental change in the individual in relation to the social-cultural environment. In Situated Learning Theory, the unit of analysis is a *community of practice*. According to this theory, learning is viewed as the progressive shift of the community from novices to masters (Lave and Wenger, 1991). The goal of this critical review is not to compare these units of analysis (the reader is referred to Engeström and Miettinen (1999) for such discussion); instead, this paper aims to add another way of viewing the learning process from an AT perspective. The concepts of *contradictions* and *activity system* are the foci in this literature review. These concepts provide the author's doctoral studies with theoretical foundations by which to address questions central to understand AT: How does the mathematics educational community define the *activity system*? How are the components of the *activity system* operationalised? And, what types of *contradictions* are being identified using this unit of analysis?

The critical review addresses: 1) the historical development of the *activity system* and 2) the various ways in which the mathematics educational community has utilised it. For these reasons, the scope of this review includes the classic papers on AT and the empirical research in the field of mathematics education that has drawn upon AT. The review is limited to those studies in mathematics education that have utilised the *activity system* as a descriptive and analytic lens. A comprehensive set of empirical studies is purposefully selected from various levels (ranging from primary to university level mathematics) to obtain a broader sense for the utilisation of the *activity system* according the mathematics educational research community. Although others have made comparisons between AT and other Neo-Vygotskian views in mathematics education, (see Monaghan, 2004) such comparisons are considered to be beyond the scope of this review.

A brief history of the unit of analysis in AT

AT has been described as a "psychological and multidisciplinary theory with a naturalistic emphasis that offers a framework for describing activity and provides a set of perspectives on practice that interlink individual and social levels" (Barab *et al,* 2004, p 199). Its unit of analysis is the *activity system* (see Figure 1). The perspectives on AT vary across disciplines such as work management, institutional education (Engeström, 2001) and human computer interaction (Kuutti, 1996). The *activity system* is the common lens that guides the analysis across multidisciplinary fields.

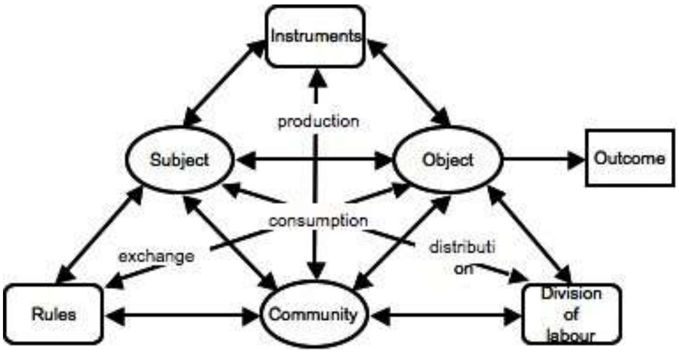

Figure 1. Activity System [Adopted from Engeström (1987)]

At the heart of AT is the concept that encapsulates the collective, object-oriented, and culturally mediated social relations of human activity, termed *activity system* by Engeström. The concepts that underpin the *activity system* are traced back from Hegelian philosophy to Marxian historical materialism (Engeström, 1999). Although many writers have contributed to develop such concepts, the focus here is on the seminal work of three main authors: Soviet psychologists L. Vygotsky and A.N. Leont'ev, and Finnish researcher Y. Engeström, whose theses are the classical building blocks from which other contributions stem.

Engeström (2001) traces back in history two previous generations of the unit of analysis by referring to the work of L. Vygotsky as first generation AT, and the work of Vygotsky's pupil, A.N. Leont'ev as second generation AT. As Wertsch (1981) rightly points out, Vygotsky did not write specifically about the concept of activity. Instead, he developed a powerful idea visualised as a triangular unit of analysis, the *mediated act* (Vygotsky, 1981), to explain human behaviour in mediated relation to its social-cultural environment (Figure 2).

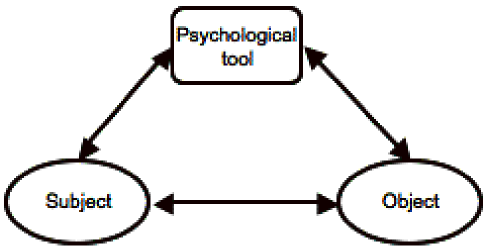

Figure 2. Mediated Act [Adopted from Vygotsky (1981)]

Building on the idea of mediation, Leont'ev (1978, 1981) conceptualised activity as composed of three different units of analysis (activity, actions, and operations). Each of the three units may be identified according to the particular psychological function following a hierarchical approach. Wertsch (1981) gives an explanation of Leont'ev's units as a hierarchical structure (see Figure 3). That is, the top of the structure is composed of activities, which are directed to achieve collective motives. In the middle are actions, which are directed to achieve individual goals, and at the bottom are operations, which are identified by the conditions in which they are carried out. In brief, operations are transformed into actions once individuals have automatised them, actions realise activities, and different activities are distinguished by their different motives.

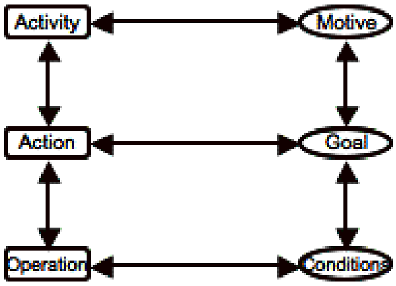

Figure 3. Hierarchical Activity Structure [Adopted from Koschmann, Kuutti and Hickman (1998)]

One of the key differences between Vygotsky and Leont'ev's generations is the differentiation of collective and individual unit of analysis. However, according to Daniels (2004) the individual-collective dichotomy that differentiated preceding units of analysis is progressed with Engeström's (1987) expanded *activity system* (Figure 3).

Engeström's expansion added the components of community, division of labour, and rules to the Vygotskian triangular 'mediated act' in order to:

> enable an examination …of macro-level of the collective and the community in preference to a micro-level concentration on the individual actor or agent operating with tools. This expansion… aims to represent the social/collective elements in an *activity system*…while emphasizing the importance of analysing their interactions (Daniels, 2004, p 123).

An analysis of the use of this *activity system* according to the interpretations given by the mathematics educational community is discussed below.

How does the mathematics educational community define the *activity system*?

The *activity system* offers educational researchers a methodological perspective in which the micro could be included within broader macro educational contexts levels, as recognised by Jaworski and Potari (2009). In order to visualise the micro-macro nested nature of educational contexts levels, I have categorised them as three concentric circles (see Figure 4).

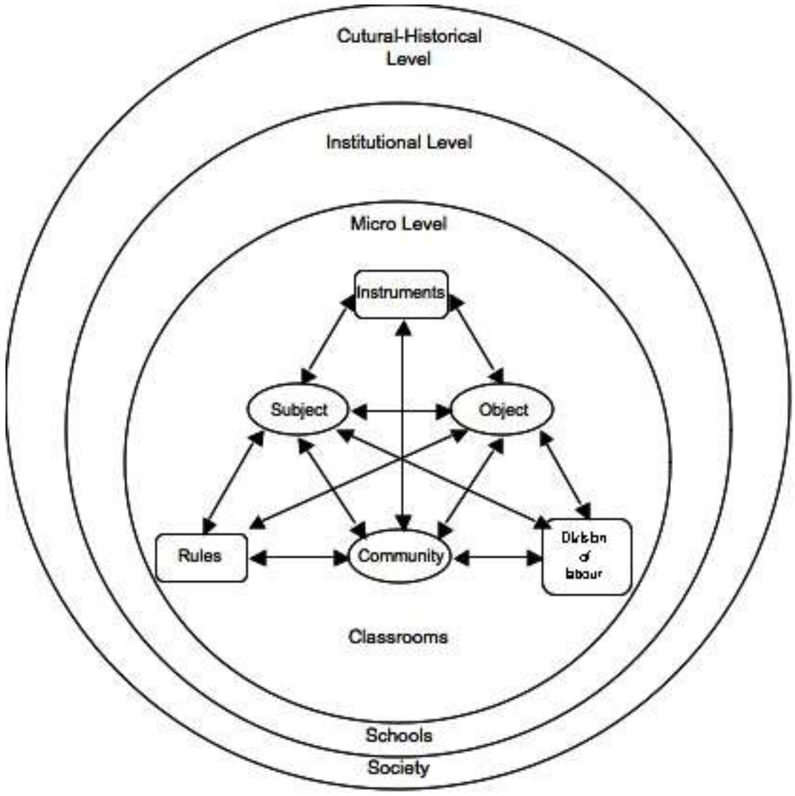

Figure 4. Nested activity systems within Educational Context Levels

The first level is the micro context, which refers to the “immediate interactional setting” (de Abreu, 2000, p 2). The micro context is nested within non-immediate second and third levels, which I denote as the institutional context and the broader cultural-historical contexts, respectively. Below, I use this nested structure to classify empirical research. I select educational studies that range from primary to university mathematics to obtain a more complete sense of how the mathematics educational research community has tended to define, design, and research the *activity system.*

My efforts to categorise research according to how *activity systems* fit in my nested educational contexts, lead to the result that few studies take advantage of being nested in other activity systems. For example, at the micro level the *activity system* is taken to be the classroom or computer laboratory (see Ho, 2007; Zevenbergen and Lerman, 2007; Hardman, 2005) and workplace settings (see Goodchild and Jaworski, 2005; FitzSimons, 2005a; FitzSimons, 2005b; Roth, 2003; Williams *et al,* 2001). At the institutional level, the *activity system* tends to be defined as departments of mathematics, schools (see Venkat and Adler, 2008; Beswick *et al,* 2007) or entire ministries of education (see Flavell, 2004; Lim and Hang, 2003). At the cultural-historical level, the *activity system* is taken to be the concepts of numeracy (see Kanes, 2001) and problem solving (see Jurdak, 2006; Jurdak and Shahin, 2001). A similar version of this nested educational context (Figure 4) is supported in the work of Lim and Hang (2003) who used nested *activity systems* across classrooms and schools. This allowed them to extend descriptions to account for the influence exerted by the larger entity of the ministry of education. Although only few studies acknowledge that the *activity system* could be considered as nested in broader levels (see Jaworski and Potari, 2009; Lim and Hang, 2003; Blanton *et al,* 2001) there is an important advantage when researchers not only considering broader contexts, but inquire into their influence; I suggest that:

- Nesting the micro *activity system* within broader contexts (see Figure 4) may provide educational researchers with further understanding of how micro contexts are influenced and dependent upon larger and powerful entities such as the institutional and cultural-historical contexts levels.

How are the components of the activity system operationalised in mathematics education?

Barab *et al* (2004) conducted a comprehensive review of the literature on the applications of AT to technology and communication studies. Their work showed how the six components of the *activity system* - subject, object, instruments, rules, community, and division of labour – are used as “buckets for arranging data collected from needs and task analyses, evaluations, and research” (p 207). These authors used the bucket-metaphor to describe how each of the components serves as a label and a category. Buckets are labelled with the name of each of the components of the *activity system* and get filled with data that are perceived by the researcher to fit that particular category. For instance, a bucket with the label of instruments tends to be filled with data that have been categorised by the researcher to describe instruments such as computers. In Jonassen and Rohrer-Murphy’s (1999) study, a five-step method is suggested to fit data into each of the six bucket-components as a way to apply AT to educational studies from a constructivist’s view. Below, I show that the mathematics educational community has employed the same bucket-application, while I emphasise that there is no agreed way within mathematics education to implement AT. Barab *et al* (2004) have already made this major point with respect to technology and communications research. I focus only on the micro context of the classroom to illustrate how these components are used as buckets, while I am taking into account that their definition depends on the aim of the research and context level.

The ***subject*** component is defined as an individual or individuals involved in the central activity, according to Engeström (1987). Some researchers have used the subject component as an analytical anchor to design the entire *activity system*. At the micro level of the mathematics classroom, the subject is referred to as a student or a group of students ranging from primary to university level (see Zurita and Nussbaum, 2007; Jurdak, 2006; Flavell, 2001; Williams *et al,* 2001). The educator(s) or educator-researcher(s) also falls into the bucket of subject in the studies that approach the research from the perspective of teachers (Hardman, 2007; Hardman, 2005; Jaworski, 2003). In those studies in which the micro context are workplaces, the subject tends to be the educational worker(s) such as the numeracy coordinator or the mathematician (see Venkat and Adler, 2008; FitzSimons, 2005a; FitzSimons, 2005b; Roth, 2003) to mention just a few.

According to Engeström, the ***object*** refers to the “raw material or problem space at which the activity is directed and which is moulded or transformed into *outcomes* with the help of physical and symbolic, external and internal *tools*” (cited in Popova and Daniels, 2004, p 196). The object-bucket tends to fill with “sense-making” or “ultimate reasons” for the subject’s behaviors (Kaptelinin, 2005, p 5). For instance, if the subject of activity is a teacher or a worker of the educational system, then the object of activity tends to be described as long term goals such as improving teaching, learning mathematical practices, or developing motivational and technical skills of students (see Venkat and Adler, 2008; Hardman, 2007; Hardman, 2005; Kanes, 2001). If the subject of activity is the student(s), then the object tends to be described as short term goals such as practicing algorithms, preparing for formative assessment, or solving assigned mathematical problems (see Zurita and Nussbaum, 2007; Jurdak, 2006; Jaworski, 2003; Roth, 2003; Flavell, 2001; Williams *et al,* 2001).

The *instruments*, also referred to as tools, are "anything used in the transformation process, including both material tools and tools for thinking" (Kuutti, 1996, p 14). There are two kinds of instruments, those that aid in the transformation of behavior (psychological tools) and those that aid in the transformation of the environment (material tools). There are studies that focus on both psychological and material tools (see Zevenbergen and Lerman, 2007; Hardman, 2005). In these, the instruments-bucket tends to include language, gestures, and group work, in conjunction with computing technologies. In addition, there are studies that focus only on psychological tools (see Jurdak, 2006; Roth, 2003; Williams *et al,* 2001). In these, the instruments-bucket tends to be filled with tools of mathematics, concepts, strategies, procedures, language, and students' mathematical knowledge. Furthermore, there are also those studies that focus only on material tools (see Coupland and Crawford, 2006; Groves and Dale, 2005). For these, the instruments-bucket is taken to include technologies such as computers, calculators, mathematics software, interactive whiteboards, and dictionary, to name a few examples.

The *community* is composed of the subject and other individuals that are brought together by a shared object (Engeström, 1996). The members of the community are usually organised to meet at a common place and time. In classrooms of mathematics, the *community* is usually represented by the teacher and students (see Jurdak, 2006; Hardman, 2005). In a larger macro context, the community-bucket tends to contain family, friends, other educational officials (Flavell, 2001) industry workers (Kanes, 2001) and policy makers (Venkat and Adler, 2008). The community binds individuals together through social rules and division of labour.

The *rules* represent norms, conventions, or social traditions that are established by the community to govern its members (Engeström, 1998). Some researchers of mathematics education differentiate between implicit and explicit rules. Implicit rules are similar to Yackel and Cobb's (1996) concept of 'sociomathematical norms' in that they are used to set permissible or impermissible behaviour in order to regulate the community's argumentation and opportunities for discussion in the mathematical classroom. Examples of implicit rules are: the gesture of raising the hand before addressing a teacher's question, as opposed to shouting out (Hardman, 2005) or speaking in English, where this is the mandated instructional and communicative language (Jurdak, 2006). On the other hand, the teacher or school authorities may set explicit rules. Examples of these include assessments (Jaworski, 2003), curriculum protocols, algorithms (Kanes, 2001), the teacher's probing and questioning (Williams *et al,* 2001), writing in the correct genre (Flavell, 2001), and whole class grouping by ability (Venkat and Adler, 2008).

The *division of labour* component refers to the splitting-up of human labour among members of the community according to its vertical and horizontal dimensions (Engeström, 1987). The horizontal dimension refers to negotiations of basic tasks and responsibilities between the community's members. Power relations and authority define the vertical dimension. Some examples that fill the bucket labelled as division of labour are the assigning of task, interventions, collaborative agreements, validation of solutions, student-centred or teacher-centred pedagogy (see Jurdak, 2006; Hardman, 2005; Jaworski, 2003; Flavell, 2001) among the members of a community that is based on hierarchies of authority ranging from students, teachers, educational coordinators, curriculum managers, etc.

The Concept of Contradictions

The concept of *contradictions* in AT is the source driving development and change of a central activity under investigation (Engeström, 2001). It follows that overcoming *contradictions* is key to change the system. *Contradictions* should not be conceptualised as everyday solvable problems. *Contradictions* in AT differs from a common-sense understanding of the word in that they are long term and systemic formations that occur within collective systems (Engeström, 2008, p 382).

> Contradictions are fundamental tensions and misalignments in the structure that typically manifest themselves as problems, ruptures, and breakdowns in the functioning of the activity system (Virkkunen and Kuutti, 2000, p 302).

Engeström (1987) defines four types of contradictions; that is, primary contradictions occur within each of the components, and secondary contradictions occur between components of a central activity system. If there are two or more activity systems, a tertiary contradiction could also occur between the objects of activity of a central activity and the object of neighbouring activities. A quaternary contradiction may occur between the rest of the components of a central activity and its neighbouring activities.

The four-level criteria set by Engeström suggests that educational researchers in mathematics have tended to identify *contradictions* in terms of the *activity system's* components; for instance, say that a secondary *contradiction* is the result of the interaction between the component of subject (a student) and a new instrument (calculator). Moreover, it is my interpretation that the potential of *contradictions* as sources of change have a positive annotation implying the possibility of change from trying to recognise and overcome them. The scope of the concepts of *contradictions* and the *activity system* is merely a descriptive one as implemented by some mathematics educational researchers. These concepts are practically used to point out areas of conflict and possible intervention with improvement purposes; however, the researcher is left to critically discern this conflict. Therefore, I can argue that *contradictions* is a powerful concept for educational researchers and a new conceptual means to describe conflicts that occur within the *activity system* set as the micro learning contexts such as mathematics classrooms. However, I consider the importance to take into account that "the burden on educational researchers is not merely to articulate principles, but to provide practitioners with tools to resolve the contradictions that arise in applying them" (Roschelle, 1998, p 250). Below, I analyse the types of *contradictions* that are identified in the educational literature utilising Engeström's four levels of contradiction approach.

What types of contradictions are identified using the activity system?

Research on the identification of *contradictions* falls into two overall categories: studies that introduce a new instrument and studies that introduce a new object into the *activity system*. Both illustrate that the introduction of a new instrument or object from the outside into the central *activity system* disrupts the dynamics between components creating secondary *contradictions* and between micro and macro context creating tertiary *contradictions*.

The results of *contradiction* identification in studies that introduce an instrument to the central activity show that researchers mainly identified secondary *contradictions* (that is between the components of an *activity system*). Zevenbergen and Lerman (2007) defined a single *activity system* (the macro context of the mathematics classroom). They found that the introduction of the instrument (interactive whiteboards) redefined all components of the *activity system* allowing them to find secondary *contradictions* (between components of subject and instruments). For instance, they described a secondary *contradiction* expressed by the subject (the teacher) as experiencing conflict with time and the interactive whiteboards (instrument) management. The authors only defined a single *activity system*; this implies that *contradiction* identification is restricted to primary and secondary *contradictions*. In a similar study, Hardman (2005) defined two interacting *activity systems* (the classroom and computer laboratory), and used them to compare across micro contexts allowing for the identification of secondary *contradictions* similar to the ones found in Zevenbergen and Lerman's study. Since Hardman's analysis was comparative in nature and did not analyse between the components of the two activity systems, then tertiary or quaternary contradictions are omitted.

Similar *secondary contradictions* between components are echoed in the studies that use AT to analyse the introduction of a *new object* into the *activity system*. For instance, Goodchild and Jaworski (2005) used a single activity system located in the macro context level (a Learning Communities in Mathematics project) to describe *contradictions* "between the stated desire to be part of the project community [object] yet an apparent distancing of themselves [the mathematics teachers] from sharing the responsibilities of the community [mathematics teachers and university researchers] as a whole [division of labour]" (p 46). As in previous studies, only the micro context is analysed giving rise to secondary *contradictions* between the components of a single *activity system*.

Others who have advanced the discussion on *contradiction* identification are Venkat and Adler (2008). These authors analysed two interacting *activity systems* to compare across the institutional context level. Venkat and Adler found secondary *contradictions* between the actions of two different subjects (numeracy coordinators) in two different mathematics departments (two interacting *activity systems*). In theory, these subjects had a common object (Key Stage 3 national policy); but in practice, the subjects implemented the national policy differently. What differentiated this study from others is the use of theoretical concepts within AT such as boundary object and boundary crossing activities, which are utilised to coordinate actions and perspectives from these two policy coordinators who started with a shared theoretical aim, but acted differently to implement it. These concepts further the understanding of the *contradictions*; however, only secondary *contradictions* are possible to identify in this study since the authors only looked between the components of each *activity system* in a comparative manner such as Hardman's (2005) study.

There is a gap in the mathematics educational literature on *contradiction* identification that results from analysing micro *activity system* nested within broader macro activity systems. Lim and Hang (2003) point the way to mathematics educational researchers in a study about integrating technology in schools. Although this study is not situated in the mathematics classroom, it offers an exemplary analysis of different types of *contradictions* that appear at the micro (classroom) and macro (school and ministry of education) contexts. Lim and Hang use the *activity system* (the classroom) nested within the institutional context (the activity system representing the school). The authors focus on the object component between two nested *activity systems*. This allowed them to state a tertiary contradiction between the objects of two nested *activity systems*. For instance, the object of the classroom (which is to provide students with 'higher order skills' in ICT) differentiates from the object of the school (which is to increase school ranking by improving examination results). It is also helpful to note that Lim and Hang perceive secondary *contradictions* as being more likely to be resolved than tertiary ones, which is perhaps due to the micro or macro context in which *contradictions* are identified.

The research studies reviewed above introduce new instruments (technology) and objects (educational project or policy) into the central system of the classroom. The types of *contradictions* that are identified by mathematics educational researchers depend on weather the researcher looks between the components of a single *activity system* or between multiple *activity systems*. The reviewed literature shows that the identification of contradictions mainly results in secondary *contradictions* (when single or dual interacting *activity systems* are analysed or compared across contexts). The concepts of boundary object and crossing activity further the understanding of the nature of *contradictions* in studies that investigate a shared object by comparing across dual interacting *activity systems*. In addition, *contradiction* identification is advanced in those studies that investigate micro learning context nested within institutional contexts. The nested nature of the activity systems allow researcher to extend their description to broader educational contexts. *Contradiction* identification is also advanced when researcher analyse between the objects of two *activity systems*, which is results in a tertiary *contradiction*. Therefore, it is important not only to analyse between activity systems in order to broader the identification of *contradictions*, but to consider the micro, institutional, and cultural-historical context to understand how these larger entities influence micro contexts.

What types of contradictions are being identified using 'gap analysis'?

In the previously stated literature review conducted by Barab *et al* (2004) on the applications of AT to technology and communication studies, the authors show that another way to carry out contradiction identification is that of "*gap analysis"* (p 208). Gap analysis is also evident in the educational literature aimed at investigating the mathematics required for competence in the workplace. This particular set of research investigates 'the gap' between college mathematics and the mathematics needed in the workplace.

Contradiction identification in the school-work literature (see Roth, 2003; Williams and Wake, 2007; Williams *et al,* 2007) is aimed at creating a mathematics curriculum that can cope with the demands of the workforce market. Usually, these studies use two pre-defined *activity systems* representing the context for learning and practicing mathematics at school and work, which are then cross-compared from the perspective of the subject (student, worker, or newcomer). *Contradictions* are not identified in terms of components as in the research reviewed in the previous section; instead, the subject "embodies the contradictions of the two systems, as do we all in general when we cross barriers" (Williams *et al,* 2001, p 79). Therefore, a primary *contradiction* that is always recurrent in such studies is phrased in terms of the subject having a double way of knowing mathematics. The theoretical mathematics learnt in school and the practical mathematics learnt at the workplace form this double knowledge. These studies provide a different way to see *contradiction*s; thus, advancing *contradiction* identification.

In the school-work gap literature, Roth (2003) focuses mainly on the interpretations of graphs by scientists (the subject) to point out that *contradictions* can arise when subjects are introduced to a new situation (unfamiliar graphs). Using an incomplete mapping of the *activity system*, Roth's evidence shows that contradictions arise "in the form of failed hypotheses, failing alignment between graphs and images, or failure to perceive photoreceptors and the corresponding absorption graphs" (p 185). This study does not look at the bottom part of Engeström's triangular unit of analysis. The operationalisation of the unit of analysis omits the rules, community, and division of labour. In this manner, the *activity system* (the workplace of laboratory) is reduced to only three components: scientists (subject), graph (object), and software, visual knowledge of interpretations (instruments). This incomplete mapping of the activity system is also seen in earlier studies (see Zevenbergen and Lerman, 2007; Groves and Dale, 2005). It is important to note that research studies that only operationalise the top three components of the activity system (subject, instruments, object) are in fact operationalising the unit of analysis in first generation AT (the mediated act), as opposed to the complete activity system in third generation AT, as discussed earlier in this review. Crucially, Groves and Dale (2005) recognise that mapping all component of third generation activity systems is necessary, stating that:

> With hindsight, it seems clear that we paid attention only to the "top half" of the activity theory triangle, without paying attention to the other components. Moreover, we had not taken into account some of the "principles" of activity theory: that activity systems should be the prime unit of analysis that an activity system always comprises multiple points of view, traditions and interest; that activity systems can only be understood against their own history; the central role of contradictions as sources of change and development; and the possibility of expansive transformations in activity systems (p 9).

The reviewed studies on the ‘gap analysis’ between mathematical knowledge learnt in school and mathematical knowledge learnt in workplaces use activity theory to compare across these two particular contexts. There is a primary *contradiction* that is embedded in the subject when she/he enters a new environment of the activity system being a new workplace or a new situation. *Contradiction* identification is purposefully done to give recommendations aimed at changing the mathematics curriculum to resemble the needs of workplaces. In some reviewed studies, only isolated triads of components are considered, whilst ignoring the rest of the components in the *activity system* (Zevenbergen and Lerman, 2007; Groves and Dale, 2005; Roth, 2003) yielding an incomplete or, in some extreme cases, no mapping of the components of Engeström’s activity system. This incomplete or total omission of the bottom concepts of the unit of analysis, in my opinion, restricts *contradiction* identification by not accounting for the *contradictions* that may rise between the bottom three components (rules, community, division of labour).

Concluding remarks

With a general objective to contribute to the field of mathematics education, I set out to conduct this literature review to critically analyse the empirical research that draws upon AT. The reviewed body of literature points to a distinction between the practical utilisation of AT by the mathematics educational community and the interventionist approach indented by Engeström (1987). This literature review leads me to focus on the particular goal to develop an AT perspective by which to analyse the mathematics learning processes that occurs in the *activity systems* of small tutorial groups at the university level. As a consequence, my research questions are as follows:

1. What does it mean to conduct mathematics educational research from an AT perspective?

2. How do objects of activity develop in the *activity systems* represented by the learning of undergraduate mathematics in small tutorial groups?

3. What components of the *activity system* come into play during interaction? How are *activity systems* transformed over time?

4. What *contradictions* emerge in and between micro (tutorials), institutional (university), and cultural-historical educational context levels (higher education system)? Are *contradictions* resolved? If so, how?

In this critical review, I focus on how the mathematics educational community has implemented the concepts of *contradictions* and the *activity system*. By combining these two powerful concepts, I have argued that the *activity system* ought to be methodologically used in ways that allow researchers to broaden the exploration of *contradictions*. The evidence presented here suggests that a complete operationalisation of the *activity system*, consisting in a conceptual mapping of all components to a learning context, is necessary. This complete conceptualisation allows researchers to define and analyse *contradictions* in terms of all six components.

In addition, the evidence shows that the analysis of *contradictions* between different objects of activity from nested activity systems allows for the identification of tertiary *contradictions*. In terms of the educational system, I suggest that *activity systems* should be nested within complex systems to accurately represent the micro, the institutional, and the cultural-historical context levels in order to broaden understanding of these powerful entities.

Furthermore, it is important to note that there is no agreed methodology in AT. The application of the *activity system* is not set, but depends on the practitioner or researcher. As suggested by Engeström and Cole (1997), the *activity system* approach "may be seen as one attempt to overcome the dualism of collective and the individually based unit of analysis" (p 304). Therefore, in order to utilise the *activity system* the researcher of education needs to consider: 1) an individual and/or collective unit of analysis, 2) a complete operationalisation of the components, 3) the identification of *contradictions* that broaden perspectives on learning, and 4) possible resolutions to *contradictions*.

Although I have not found examples of empirical research within the mathematics educational community that encapsulate full explanations of the above four points, I refer the reader to the theoretical research in other fields that may serve as a guide to understand the different tendencies in AT. For instance, Daniels (2004) is a very clear example to illustrate point one when remarking that Engeström's *activity system* is collective; that is, it refers to social practices of individuals and organizations, while recognising that it is methodologically difficult to capture evidence from the community, rules, and division of labour. Another theoretical example that may help to illustrate points three and four is the work of Warmington (2005), who argues that the study of *activity systems* should be the study of *contradictions* located within labour power (re)production. With respect to point two, I have mentioned earlier in this review that the study by Lim and Hang (2003) provides an exemplary approach to the practical applications of AT within the educational community. What is seminal about the uses of AT is that this "is an evolving framework which needs to be developed further as it is applied in empirical studies" (Engeström, 2008, p 382), encouraging the new researcher to its exploration.

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